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A Brief History of ‘Square of Opposition’

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Abstract:

In 4th century B.C, Aristotle introduced this new branch of knowledge called ‘Logic’. The main aim of this research paper is how has the notion of the ‘Square of Opposition’ developed in time to time? In this context, I will discuss how Aristotle, Apuleius, Boethius and Parsons have explained the notion of ‘Square of Opposition’. The difference in their viewpoints regarding the opposition relation will also be discussed. It will also be discussed Apuleius’ four opposition relations and Parsons’ notion of ‘Square of Opposition’. Further, it will also be discussed why the modern logicians have refuted the notion of ‘Traditional Square of Opposition’ and given the idea of ‘Cross’ of Opposition. Emphasis will also be given on how the modern logicians with the help of empty term and George Boole with the notion of ‘Existential Import’ have refuted the three opposition relations (Contrary, Subcontrary and Subaltern). It will also be discussed why Modern logicians opined that from two universal propositions, one particular proposition cannot follow. In this context, how they have proved the case with the help of Euler and Venn-Peirce diagram will be shown.

Keywords: Proposition, Opposition Relation, Empty Term, Existential Import, Diagram.

Introduction: Logic is an important topic of discussion in both Indian and Western traditions from very ancient times. In Logic, various notions have been discussed. Among them ‘Square of Opposition’ is a significant topic. According to Aristotle, we can express all the descriptive sentences through A, E, I, and O propositions. Now, these propositions are explained by examples as follows: Universal affirmative proposition (A): All men are mortal; Universal negative proposition (E): No men are mortal; particular affirmative proposition (I): Some men are mortal; particular negative proposition (O): Some men are not mortal. Now, these four propositions can represent or express through a Square.

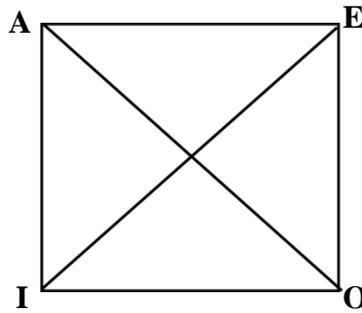


Fig-1

“There are certain logical relations between the four types of categorical proposition, when they have the same subject and predicate terms. The diagrammatic representation of these relations among these propositions is called the Traditional Square of Opposition”¹. So, it can say that there is an opposition relation between any two propositions. The opposition relation is explained as follows -

Opposition relation means a relation depicting any two propositions having the same subject and predicate terms that may (obviously) differ from each other in quality, or in quantity or in both. Any such kind of difference has been traditionally called opposition relation. Here, it can see that there are four types of opposition relations among the four types of propositions. These are contrary; sub-contrary; subaltern; and contradictory. Now, in this respect I will discuss what Aristotle wanted to say about the opposition relation.

Aristotelian view of Opposition: Among the above mentioned four types of opposition relation, Aristotle considered only two opposition relations. He did not accept (consider) the sub-contrary and subaltern opposition relation. Because according to him whatever may be considered as an opposition relation between two propositions, the second is always the denial of the first. But in the sub-contrary and subaltern oppositions, there is no denial of the other. So, he did not consider the relation between ‘A – I’ and ‘E – O’ propositions and ‘I - O’ propositions as opposition relations. As per his view two propositions are opposed as contradictories when they cannot both be true and cannot both be false, this means that if one proposition is true then the other will be false and vice versa. Again, two propositions are contraries only when they cannot both be true but may both be false. Now we can construct a square of opposition where only two opposition relations are shown by example according to Aristotle’s view²:

¹ S. S. Sharma, 2012, p. 175

² Kneale, W., & Kneale, M. (1962). *The development of Logic*, P. 54-56.

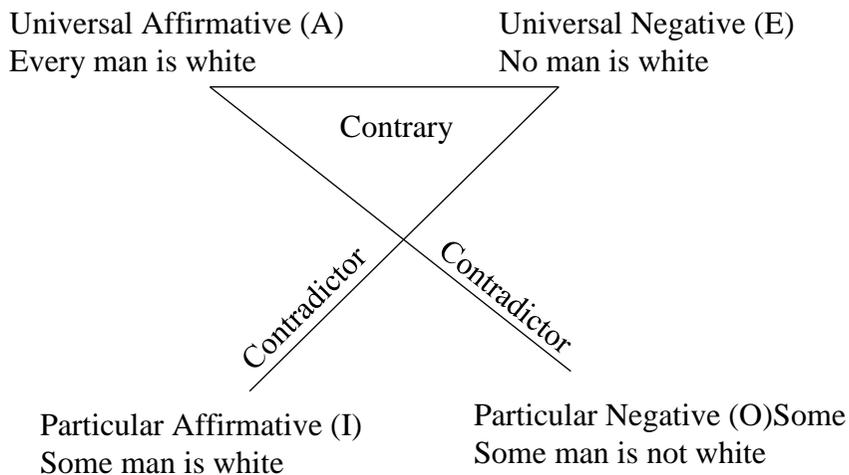


Fig-2

Later logicians say that we can easily infer sub-contrary and subaltern opposition relations. In the sub-contrary relation, two propositions cannot both be false but may both be true. This means that, in this relation, if one proposition is false then another proposition will be true. They also say that each universal proposition entails its subaltern although Aristotle does not use these expressions.

Apuleius' Logical Relations: Lucius Apuleius has explained the opposition relations from other aspects. He mentions in detail the logical relations that hold between the four types of categorical propositions in his commentary on *De Interpretatione*. But he never drew any diagrams to express the logical relation. Now, what did he say about opposition relations? According to Apuleius, an opposition relation is not only determined by truth value or denial of each other, it is determined by dissimilarities of quality or quantity or both quality and quantity between two propositions which should also be opposition. According to Apuleius, above mentioned dissimilarities entail contraries, sub-contraries, subalterns and contradictories. Now, each of the opposition relation shall be examined to find out the kind of dissimilarities each opposition contains.

- **Contrary Opposition relation:** 'A' and 'E' propositions have the contrary opposition relation. They have dissimilarities both in quality and truth-values. Because of the difference in quality, if a proposition is true then the other one is false and if one is false then the other one is uncertain (uncertain means, may be true or false).
- **Sub-contrary opposition relation:** This relation holds between 'I' and 'O' propositions. This opposition is only based on quality. In this relation, two propositions cannot be false together but can be true together.
- **Subaltern opposition relation:** This relation is in pairs- there are two pair of propositions i.e., A – I and E – O. These pair of oppositions only contain

dissimilarity in quantity. Here, between the two propositions if A proposition is true then I proposition is also true and if I proposition is false then A proposition will also be false. These are Same as E and O propositions.

- **Contradictory opposition relation:** A – O and E – I pairs have the contradictory opposition relation. Here, dissimilarity is in two types i.e., quality and quantity. In this relation, if a proposition is true then the other will be false and vice-versa.

Boethius’ Square of Opposition: Anicius Manlius Severinus Boethius also explained the opposition relation which is based on Apuleius’s explanation. He expressed that the logical relations among these propositions with the help of a diagram based on the formulations of Apuleius known as the ‘square of Opposition’. He has given a diagram of the Square of Opposition in Latin.

Parsons’ Square of Opposition: Parsons’ representation of the Traditional Square of Opposition is most widely used and referred in logical texts now. Here, the four types of logical relations are shown. They are contrary, sub-contrary, subaltern, and contradictory. Parsons’ diagrammatic representation is shown in figure (3).

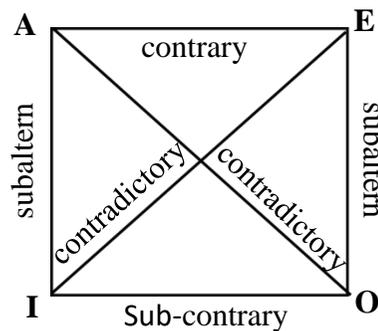


Fig-3

The explanation of Parsons square of Opposition is known as ‘Traditional Square of Opposition’. Here, we get four types of opposition relations. On the basis of the Square of Opposition, how can the truth-value of a proposition be obtained with the help of another proposition’s truth-value depending on opposition relation? Now the four types of opposition relation are explained as follows³:

- 1) **Contrary Opposition Relation:** Two universal propositions with same subject and predicate terms will be contrary to each other if they are of different quality. Between A and E proposition we see this opposition relation. For example: ‘All men are wise’ (A) and ‘No men are wise’ (E). If two propositions hold contrary relation between them, then those two propositions cannot be true together but can be false together. ‘All men

³ <http://plato.stanford.edu/entries/square/>
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are wise' (A) – if this proposition is true, then 'no men are wise' (E) cannot be true and vice versa. But these two propositions can be false at the same time.

- 2) **Sub-contrary Opposition Relation:** If two propositions have the same subject and predicate and they do not differ in terms of quantity but differ in term of quality, then there will be the relation of sub-contrary opposition between the two propositions. I and O propositions have sub-contrary opposition. Now, if two propositions have sub-contrary opposition relation between them, then they cannot be false together but can be true together. For example, "some men are mortal" (I) – if this proposition is false then "some men are not mortal" (O) – this proposition cannot be false, but these two propositions can be true together.
- 3) **Subaltern Opposition Relation:** In case of subaltern opposition if two propositions have the same subject and predicate and they do not differ in terms of quality but differ in term of quantity, then there will be the relation of subaltern opposition between them. A – I and E - O proposition have subaltern relation between them. In subaltern opposition, we can state – if the A proposition is true then the I proposition will also be true and if the E proposition is true then the 'O' proposition will also be true. In that case we can say that if the universal proposition is true then the particular proposition will also be true.
- 4) **Contradictory Opposition Relation:** The fourth type of opposition relation is the contradictory relation. If two propositions having the same subject and predicate; differ both in terms of quality and quantity then the relation between these two propositions can be termed as the relation of contradictory. A – O and E - I propositions have contradictory opposition relation between them. If one of the two propositions is true then the other one will be false and if one of the two propositions is false then the other one will be true. For an example – if 'All men are wise' this proposition is true, then 'Some men are not wise' is false and vice versa. Again, if 'No men are wise' is true then 'Some men are wise' is also false and vice versa.

So, from the Parsons Square of Opposition, it can be said that the four-opposition relation among the four propositions are completely defined.

Some Objections about Traditional Square of Opposition: The modern logicians say that contrary, sub-contrary and subaltern opposition relations do not hold. They accept only contradictory opposition. Now the question is - why do they not consider those three opposition relations? In this context, they give two reasons stated as follows:

Firstly, if a proposition does not have Existential Import, then from such proposition we cannot infer other propositions. In that context, we cannot infer the truth value of other proposition from the truth value of one proposition.

Secondly, if a term used by any proposition as subject or predicate, is an empty term, then from that proposition we cannot infer any other proposition.

I: Some S is P is symbolized as $(\exists x) (Sx \cdot Px)$
 O: some S is not P is symbolized as $(\exists x) (Sx \cdot \sim Px)$

Here, modern logicians have used some notations, such as universal quantifier (x); particular quantifier ($\exists x$); logical connectives (\supset , \cdot , \sim); and antecedent (Sx); consequent (Px). In the case of universal proposition there are two parts that is antecedent and consequent. But in particular propositions there are two conjuncts (subject and predicate). According to modern logicians, universal proposition will be true if its antecedent is false or its consequence is true. And a universal proposition will be false only when the antecedent is true and the consequence is false. Again, in case of conjunctive proposition, a proposition is true if and only if both the conjuncts are true and false otherwise. Now, in respect of that explanation, I will try to show why modern logicians did not accept those opposition relations (contrary, sub-contrary, and subaltern).

Firstly, we know that in the Square of Opposition, A and E propositions have contrary opposition relation. In this relation, it is said that, two propositions cannot be true together but can be false together.

But modern logicians say that A and E propositions can be true together, if their subject terms are empty. Let us now suppose that the subject term of A and E proposition are empty, then the truth value of subject term is false and these two propositions (A, E) are vacuously true. For an example if $(x) (Sx \supset Px)$ (A) and $(x) (Sx \supset \sim Px)$ (E) have both the antecedent are empty terms then the truth value of the antecedent is false. Then both of the propositions will be true. If both A and E propositions are true then the contrary opposition relation does not hold.

A: $(x) (Sx \supset Px)$ E: $(x) (Sx \supset \sim Px)$
 F T F T

According to Boolean interpretation, universal proposition (A, E) does not have existential import. That is through utterance of universal propositions the real existence of the subject term is not referred. We cannot ascertain truth or falsity of the subject terms of such proposition. So, it is meaningless to say that in contrary opposition relation both propositions cannot be true. Universal proposition can be true which contains terms that have no real existence. So, modern logicians do not admit contrary opposition relation between A and E proposition.

Secondly, we have seen that sub-contrary opposition relation exists between particular affirmative (I) and particular negative (O) proposition. In this relation, it is said that if one proposition is false then another proposition will be true. But modern logicians hold that no sub-contrary relation can be admitted between I and O propositions, because in this relation two propositions can be false together. For an example:

I: Some chimera is animal $(\exists x) (Sx \cdot Px)$
 O: Some chimera is not animal $(\exists x) (Sx \cdot \sim Px)$

If the subject terms of the above-mentioned forms of proposition are false then these propositions can be false. 'Chimera' is something which is unreal in the world, hence it is false and if the subject term is false then I and O proposition will become false, which violate the sub-contrary opposition relation.

Moreover, when it is said that 'Some chimera is animal' (I) – this proposition is false that means 'Some chimera is not animal' (O) – this proposition must be true, because in sub-contrary opposition, it is said that if one proposition is false, the other one will be true. But here, if the proposition: 'Some chimera is not animal' (O) is true then the question arises, which chimera is not animal? If the particular negative proposition is true, then its subject term must have a reference. But the subject term of the proposition 'Some chimera is not animal' is an 'empty term' because it does not refer to anything at all and does not refer to any animal. So, this proposition also becomes false. Therefore, it can be said that, in sub-contrary opposition relation, two propositions can be false, that is violated of sub-contrary opposition relation.

Thirdly, in traditional Square of Opposition, it is said that subaltern opposition relation holds between 'A – I' and 'E – O' propositions. In this relation it is said that if universal proposition (A) is true then the particular proposition (I) will also be true. And if 'I' proposition is false then 'A' proposition will also be false. 'E – O' proposition have same fact.

The modern logicians show that there can be no subaltern opposition relation between 'A' and 'I' propositions because it cannot be said that truth of 'A' proposition entails truth of 'I' proposition. If the subject term of 'A' proposition is empty then that is false. If the subject term (antecedent) is false then the whole proposition is true because 'A' proposition is always in a conditional form. Whereas, if that empty term appears in the subject term of 'I' proposition, then the whole 'I' proposition will be false. 'I' proposition is always in the form of conjunction where falsity in any one of the conjunct leads to the falsity of the whole 'I' proposition.

<p>A: (x) (Sx \supset Px) L.F. Sa \supset Pa F T</p>	<p>I: (\existsx) (Sx . Px) L.F. Sa . Pa F F</p>
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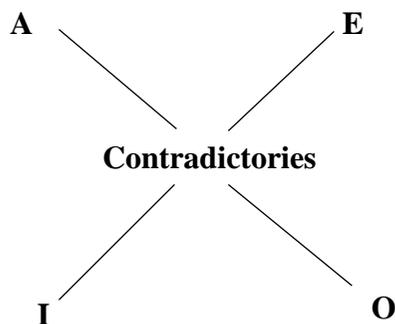
Again, if the notion of existential import of proposition is admitted then inference by subaltern opposition cannot be called a valid inference. By inference of subaltern opposition, the particular proposition entails from the universal proposition. For example –

- A: All men are honest
- I: Some men are honest

In this inference, the fallacy of existential import has been committed. So, according to modern view of subaltern opposition, the relation is not accepted between two propositions (A and I). and same problem will happen another pair of subaltern opposition (E – O).

From the above discussion, we find that the relations of contrary, sub-contrary and subaltern opposition are all falsified. The only relation in the Traditional Square of Opposition, which holds in accordance with the modern point of view, is that of contradictories between universal affirmative and particular negative as well as universal negative and particular affirmative. In contradictory relation, it is said that if one proposition is true then the other proposition will be false and vice-versa. This relation holds true in all instances and is never violated. Between A and O proposition, if the subject term of A proposition is an empty term, then the subject term will be false, and in that case 'O' proposition will be true, and in 'O' proposition the empty term appears in the subject term and makes it false and vice versa. So, the modern logicians accept only contradictory opposition relation, they do not accept the other three opposition relations. They represent this contradictory opposition relation as a 'cross' of opposition and this 'cross' of opposition is known as the modern Square of Opposition.

The Modern Square of opposition: Modern logicians accept only contradiction as a valid form of relation among the propositions of A – O and E – I. The Square of Opposition as revised by modern logicians is as follows:



Modern Square of Opposition

Fig - 4

Here, modern logicians says that logical relations among the propositions are represented by the Traditional Square of Opposition which are based only on the assumptions that the terms of the various categorical propositions are non-empty. But in the modern Square of Opposition, there is no problem of empty terms and existential fallacy. They also say that many of the immediate inferences do not hold if we consider in the context of Aristotelian system of deductive inference. Now, considering the immediate inference of conversion by limitation, we cannot draw the particular conclusions from the universal propositions. For example:

All unicorns are horses (universal premise)

Therefore, some horses are unicorns (particular conclusion)

The premise, 'all unicorns are horses' being a universal proposition is devoid of existential import and understood as a conditional statement 'if anything is a unicorn, it will be a horse'. But we know that 'unicorns' is an empty term and it does not refer to anything. If any terms do not refer to anything then it is false. So, 'all unicorns are horses'- this proposition is true. But the conclusion, 'some horses are unicorns' is false, as there is nothing which is both 'a horse and a unicorn'. So, this is an invalid inference, because we draw a false conclusion from the true premises.

Again, the modern logicians say that if the universal proposition is evacuated of their existential import, then some of the arguments are considered valid in the context of Aristotelian system. Now, consider the following as:

All Cows are Mammals.
All Cows are Herbivorous.

Therefore, some Mammals are Herbivorous.

This argument is a valid mood in 3rd figure. This valid mood name is Darapti. In this syllogism, the premises and the conclusion are true. In this context, there is no violating of any Aristotelian rules of inference. But modern logicians have shown that this valid argument is invalid by considering another argument in Darapti. In this case, the premises are true and the conclusion is false. Consider, as example:

All trespassers are prosecuted.	$\forall x (Tx \rightarrow Sx)$	
All trespassers are people.	$\forall x (Tx \rightarrow Px)$	(Sx: prosecuted)
Therefore, some people are prosecuted.	$\therefore \exists x (Px \cdot Sx)$	

From the above argument, it can be said that there are no trespassers in both the premises. In spite of, these premises turn out to be true vacuously. So, the conclusion, 'some people will be prosecuted' will be false, where nobody trespasses. Hence, it is an invalid argument.

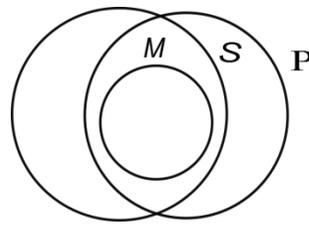
So, from the above discussion, we can say that the argument (AAI-3) is valid in the context of Aristotelian system but invalid in the modern context. For this reason, modern logicians say that Aristotelian rules of syllogistic inference are not sound, for which some arguments are invalid. So, modern logicians suggested additional rules of syllogistic inference. This rule which pertains to the quantity of the premises and the conclusion may be stated as follows:

Additional Rules: 'No valid syllogism with particular conclusion has two universal premises.'

The principle above could be seen easily with the help of Euler circles and Venn-Peirce diagrams, where some argument is valid in Euler circle but invalid in Venn-Peirce diagram. For example: The argument (AAI-3) is valid in Euler circles. Now, I will show this argument as follows:

As we know, Darapti i.e., AAI-3 is written as

All M is P
 All M is S
 ∴ Some S is P

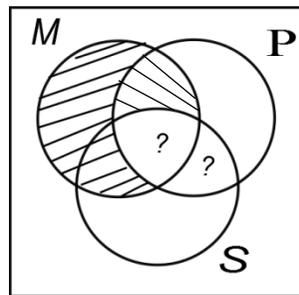


Euler diagram

In this diagram, the first premise 'all M is P' is drawn as a circle M inside P and second premise 'all M is S' is drawn as a circle M inside S. Here, it is not possible to draw S intersecting M without S intersecting P. So, we can see in this diagram, some part of circle S, is inside circle P. The conclusion can claim this fact. So, in this context this argument (AAI-3) is valid. Now, I will draw Venn-Peirce diagram as:

All M is P
 All M is S

 ∴ Some S is P



Venn-Peirce diagram

In the above diagram, the first premise 'all M is P' shown by shading off the part of M which is not P and the second premise 'all M is S' shown by shading off the part of M, which is not S. The conclusion of the argument states, 'Some S is P' which means that we must get an 'X' mark in the common part of S and P. This does not happen. Hence, a question mark is drawn (?) to show the inability of the premises to imply the conclusion. Therefore, it is said that according to Venn-Peirce Diagram, AAI-3 is an invalid argument.

Conclusion: Above the discussion, it has seen that how Aristotle, Apuleius, Boethius and Parsons have explained the notion of 'Square of Opposition. Aristotle considered only two opposition relations. He did not accept (consider) other two opposition relations. Apuleius considered four types of opposition relations on the basic of dissimilarities of quality or quantity or both quality and quantity between two propositions not only determined by truth value or denial of each other. But he never drew any diagrams to express the logical relation. Boethius also explained the opposition relation which is based on Apuleius's explanation. He has given a diagram of the Square of Opposition in Latin. Finally, it has seen Parsons' Square of Opposition is a clearest diagrammatic representation of four opposition relations which is called 'Traditional Square of Opposition'. But Modern Logicians did not consider these four types of opposition relation on the basic of 'Existential Import' and 'Empty Term'. They considered only contradictory opposition relation. They also explained the argument Darapti (AAI-3) is valid when we check its

validity as per the Euler Circle whereas it is an invalid Venn-Peirce diagram. Euler Circle does not consider the rule of existential fallacy whereas Venn-Peirce Diagrams does consider the above rule and thus Darapti (AAI-3) is invalid. For this reason, modern logicians say that, 'No valid syllogism with particular conclusion has two universal premises.

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